

ColaCorp

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Outline

Launch

Thinking

Q1

Q2

LACCIISIONS

E1

E2

Wh

Contac Refs

Would You Like a Cold Beverage with Your Inquiry?

Brian P Katz (BK)

MathFest, Tampa, FL

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Thinking Q1 Q2

Extension

E1 E2

Wh

Contact Refs

- Cola Corporation, an opportunity for inquiry (Interviewing M)
- Student thinking
- 3 M's Question, extensions of the inquiry
- Student thinking
- **5** Why use this activity



Cola Corporation: Launch

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Extension E1 E2

Why

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The Cola Corporation packages their soda in the following sizes:

- Individual cans
- Packs, consisting of 6 individual cans
- Cases, consisting of 6 packs
- Palettes, consisting of 6 cases
- Crates, consisting of 6 palettes
- Flotillas, consisting of 6 crates
- Other sizes are analogous but not named yet.

When making a shipment, the *Cola Corporation* ships the <u>exact</u> order using the largest possible (full) packages without going over the order.



Cola Corporation: Questions to Explore

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Extension: E1 E2

Wh

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- (1) CSULB submits an order for 5,000 cans. How will it be packed for shipping?
- (2) CSULB calls the *Cola Corporation* to say that the previous order was an error and should be doubled. How will the new order be packed for shipping? Answer this question using at least two different strategies and check that they agree.
- (3) LBCC has a standing order with the *Cola Corporation* that whenever an order of size S is shipped to CSULB, an order of size S/3 is shipped to LBCC. How will the order be packed for shipping?



Question 1

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Why

Contac Refs (1) CSULB submits an order for $5,000\ \mathrm{cans}.$ How will it be packed for shipping?

Question 1: Strategy 1

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Q1

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Extension

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Why

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Repeated/Block subtraction:

- \bullet 5000 3 * 6⁴ = 1112 < 1296 = 6⁴, so 3 crates
- \blacksquare 1112 5 * 6³ = 32 < 216 = 6³, so 5 palettes
- $32-0*6^2=32<36=6^2$, so 0 cases
- $32-5*6^1=2<6^1$, so 5 packs, and
- 2 cans

Question 1: Strategy 2

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Integer part:

- $5000/6^4 \approx 3.86$, so 3 crates with 1112 cans left
- $1112/6^3 \approx 5.12$, so 5 palettes with 32 cans left
- $32/6^2 \approx 0.89$, so 0 cases
- \blacksquare $32/6^1 \approx 5.33$, so 5 packs, and
- 2 cans

Question 2

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Why

Contac Refs (2) CSULB calls the *Cola Corporation* to say that the previous order was an error and should be doubled. How will the new order be packed for shipping? Answer this question using at least two different strategies and check that they agree.

Question 2: Strategy 1

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Q1 Q2

Extension:

E2 E3

Conta

Convert 10,000:

- 10000 1 * 7776 = 2224 < 7776, so 1 flotilla
- ightharpoonup 2224 1 * 1296 = 928 < 1296, so 1 crate
- 928 4 * 216 = 64 < 216, so 4 palettes
- \bullet 64 1 * 36 = 28 < 36, so 1 case
- 28 4 * 6 = 4 < 6, so 4 packs, and
- \blacksquare 4 cans

Question 2: Strategy 2

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Extensions

E2 E3

Conta

Double the packages:

$$\begin{split} 2\left(\frac{2}{6^0},\frac{5}{6^1},\left(\frac{0}{6^2}\right),\frac{5}{6^3},\frac{3}{6^4}\right) \\ &= \frac{4}{6^0},\frac{10}{6^1},\left(\frac{0}{6^2}\right),\frac{10}{6^3},\frac{6}{6^4} = \frac{4}{6^0},\frac{10}{6^1},\left(\frac{0}{6^2}\right),\frac{10}{6^3},\frac{0}{6^4},\frac{1}{6^5} \\ &= \frac{4}{6^0},\frac{10}{6^1},\left(\frac{0}{6^2}\right),\frac{4}{6^3},\frac{1}{6^4},\frac{1}{6^5} = \frac{4}{6^0},\frac{10}{6^1},\frac{0}{6^2},\frac{4}{6^3},\frac{1}{6^4},\frac{1}{6^5} \\ &= \frac{4}{6^0},\frac{4}{6^1},\frac{1}{6^2},\frac{4}{6^3},\frac{1}{6^4},\frac{1}{6^5} \end{split}$$

Question 2: Strategy 3

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Extend the stacked algorithm:

	1	3^1	5	0^{1}	5	2
*						2
	1	1	4	1	4	4

Question 3

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Q1 Q2

Extension

E1 E2 E3

Why

Contac Refs (3) LBCC has a standing order with the *Cola Corporation* that whenever an order of size S is shipped to CSULB, an order of size S/3 is shipped to LBCC. How will the order be packed for shipping?

Question 3: Strategy 1

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Q2 Q3

Extension

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In base-6, $\frac{1}{3} * x = (\frac{1}{6} * x) * 2$:

$$\frac{1}{3} \left(\frac{1}{6^5}, \frac{1}{6^4}, \frac{4}{6^3}, \frac{4}{6^2}, \frac{4}{6^1}, \frac{4}{6^0} \right) = \frac{2}{6^4}, \frac{3}{6^3}, \frac{2}{6^2}, \frac{3}{6^1}, \frac{3+\frac{1}{3}}{6^0}$$
$$= \frac{2}{6^4}, \frac{3}{6^3}, \frac{2}{6^2}, \frac{3}{6^1}, \frac{3}{6^0}, \frac{2}{6^{-1}}$$

Question 3: Strategy 2

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Extending long division:

2	2	23	23233_{6}
$3\overline{)114144_6}$,	$3\overline{)114144_6}$	$3\overline{)114144_6}, \dots$	$3\overline{)114144}_{6}$
10	10	10	10
	14144_{6}	14144_{6}	14144_{6}
	_	<u>13</u>	<u>13</u>
		$\overline{1144}_{6}$	-1144_{6}
			<u>10</u>
			144_{6}
			<u>13</u>
			14_{6}
			$\frac{13}{1_6}$
			1_6

Question 3: Strategy 2

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Extending long division:

```
23233.2_{6}
3\overline{)114144.0_{6}}
   \frac{10}{14144}_6
      \frac{13}{1144_6}
         10
           \overline{1}44_{6}
         \frac{13}{146}
              13
                 \overline{1.0_{6}}
                 1.0
```



Computational Technology

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Extension

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```
convert (10000/3) to base 6
A NATURAL LANGUAGE ST MATH INPUT
                                                         ■ EXTENDED KEYBOARD 

EXAMPLES 

UPLOAD 

RANDOM
 Input interpretation
 convert \frac{10000}{3} to base 6
 Result
 23233.26
 Other base conversions
                                                                        Show digit key
                                                                                          More digits
                                                                                                        More bases
 1.1010000010101..._2 \times 2^{11}
 310011.111111111...4
 6405.2525252525...8
 1b19.4<sub>12</sub>
 d05.55555555555...16
```



Extensions!

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Q1

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Extensions

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Contac[.] Refs What does this make you wonder?

What does this make you want to explore?

M's Frustration/Conjecture

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$0.\overline{3}_{10} = \frac{1}{3} = 0.2_6$

Question

How can we tell when a fraction will have a finite or infinite representation? More specifically, what properties of n and b guarantee a finite (or infinite) representation of $\frac{1}{n}$ in base-b?



E1: (In)finite dec/heximals

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Extension E1

E3 Why

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$base \rightarrow$	2	3	4	5	6	7
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1/2	0.12	$0.\overline{1}_{3}$	0.24	$0.\overline{2}_{5}$	0.3_{6}	$0.\overline{3}_{7}$
1/3	$0.\overline{01}_{2}$	0.13	$0.\overline{1}_{4}$	$0.\overline{13}_{5}$	0.26	$0.\overline{2}_{7}$
1/4	0.01_{2}	$0.\overline{02}_{3}$	0.14	$0.\overline{1}_{5}$	0.136	$0.\overline{15}_{7}$
1/5	$0.\overline{0011}_{2}$	$0.\overline{0121}_{3}$	$0.\overline{03}_{4}$	0.15	$0.\overline{1}_{6}$	$0.\overline{1254}_{7}$
1/6	$0.0\overline{01}_{2}$	$0.0\overline{1}_{3}$	$0.0\overline{2}_{4}$	$0.\overline{04}_{5}$	0.16	$0.\overline{1}_{7}$
1/7	$0.\overline{001}_{2}$	$0.\overline{010212}_{3}$	$0.\overline{021}_{4}$	$0.\overline{032412}_{5}$	$0.\overline{05}_{6}$	0.17

Theorem (M's Conjecture)

For integers n and b greater than 1, the fraction $\frac{1}{n}$ has a finite representation in base-b if an only if n divides a power b. Moreover, the length of the finite representation (number of digits in the representation before the start of a tail of all 0s) is the smallest power ℓ such that $n|b^{\ell}$.

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$base {\to}$	2	3	4	5	6	7
1/2	0.12	$0.\overline{1}_{3}$	0.2_{4}	$0.\overline{2}_{5}$	0.36	$0.\overline{3}_{7}$
1/3	$0.\overline{01}_2$	0.13	$0.\overline{1}_{4}$	$0.\overline{13}_{5}$	0.26	$0.\overline{2}_{7}$
1/4	0.01_2	$0.\overline{02}_{3}$	0.14	$0.\overline{1}_{5}$	0.13_{6}	$0.\overline{15}_{7}$
1/5	$0.\overline{0011}_{2}$	$0.\overline{0121}_{3}$	$0.\overline{03}_4$	0.15	$0.\overline{1}_{6}$	$0.\overline{1254}_{7}$
1/6	$0.0\overline{01}_{2}$	$0.0\overline{1}_{3}$	$0.0\overline{2}_{4}$	$0.\overline{04}_{5}$	0.16	$0.\overline{1}_{7}$
1/7	$0.\overline{001}_{2}$	$0.\overline{010212}_{3}$	$0.\overline{021}_{4}$	$0.\overline{032412}_{5}$	$0.\overline{05}_{6}$	0.17

- $\frac{1}{b} = 0.1_b$

2

 0.1_{2}

 0.01_{2}

 0.01_{2}

 $0.\overline{0011}_{2}$

 $0.0\overline{01}_{2}$

 $0.\overline{001}_{2}$

3

 $0.\overline{1}_3$

 0.1_{3}

 $0.\overline{02}_{3}$

 $0.\overline{0121}_{2}$

 $0.0\overline{1}_{3}$

 $0.\overline{010212}_{3}$

4

 0.2_{4}

 $0.\overline{1}_{4}$

 0.1_{4}

 $0.\overline{03}_{4}$

 $0.0\overline{2}_{4}$

 $0.\overline{021}_{4}$

5

 $0.\overline{2}_{5}$

 $0.\overline{13}_{5}$

 $0.\overline{1}_{5}$

0.15

 $0.\overline{04}_{5}$

 $0.\overline{032412}_{5}$

6

 0.3_{6}

 0.2_{6}

 0.13_{6}

 $0.\overline{1}_{6}$

 0.1_{6}

 $0.\overline{05}_{6}$

 $0.\overline{3}_{7}$

 $0.\overline{2}_{7}$

 $0.\overline{15}_{7}$

 $0.\overline{1254}_{7}$

 $0.\overline{1}_{7}$

0.17

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Theorem

 $base \rightarrow$

1/2

1/3

1/4

1/5

1/6

1/7

$$\frac{1}{b} = 0.1_b$$
; $\frac{1}{b^2} = 0.01_b$

$$\frac{1}{b-1} = 0.\overline{1}_b$$

$$\frac{1}{b+1} = 0.\overline{0(b-1)}_b$$

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Extension

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$base{}{}{}{}{}{}{}{}{}{}{}{}$	2	3	4	5	6	7
1/2	0.12	$0.\overline{1}_3$	0.2_{4}	$0.\overline{2}_{5}$	0.36	$0.\overline{3}_{7}$
1/3	$0.\overline{01}_{2}$	0.1_{3}	$0.\overline{1}_4$	$0.\overline{13}_{5}$	0.26	$0.\overline{2}_{7}$
1/4	0.01_2	$0.\overline{02}_{3}$	0.1_{4}	$0.\overline{1}_{5}$	0.136	$0.\overline{15}_{7}$
1/5	$0.\overline{0011}_{2}$	$0.\overline{0121}_{3}$	$0.\overline{03}_{4}$	0.1_{5}	$0.\overline{1}_{6}$	$0.\overline{1254}_{7}$
1/6	$0.0\overline{01}_{2}$	$0.0\overline{1}_{3}$	$0.0\overline{2}_{4}$	$0.\overline{04}_{5}$	0.16	$0.\overline{1}_{7}$
1/7	$0.\overline{001}_{2}$	$0.\overline{010212}_{3}$	$0.\overline{021}_{4}$	$0.\overline{032412}_{5}$	$0.\overline{05}_{6}$	0.17

- $\frac{1}{b} = 0.1_b$; $\frac{1}{b^2} = 0.01_b$
- $\frac{1}{b-1} = 0.\overline{1}_b$; What if b = 2?

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$base{}\!\to$	2	3	4	5	6	7
1/2	0.12	$0.\overline{1}_{3}$	0.24	$0.\overline{2}_{5}$	0.36	$0.\overline{3}_{7}$
1/3	$0.\overline{01}_{2}$	0.13	$0.\overline{1}_{4}$	$0.\overline{13}_{5}$	0.26	$0.\overline{2}_{7}$
1/4	0.01_2	$0.\overline{02}_{3}$	0.14	$0.\overline{1}_{5}$	0.136	$0.\overline{15}_{7}$
1/5	$0.\overline{0011}_{2}$	$0.\overline{0121}_{3}$	$0.\overline{03}_4$	0.15	$0.\overline{1}_{6}$	$0.\overline{1254}_{7}$
1/6	$0.0\overline{01}_{2}$	$0.0\overline{1}_{3}$	$0.0\overline{2}_{4}$	$0.\overline{04}_{5}$	0.1_{6}	$0.\overline{1}_{7}$
1/7	$0.\overline{001}_{2}$	$0.\overline{010212}_{3}$	$0.\overline{021}_{4}$	$0.\overline{032412}_{5}$	$0.\overline{05}_{6}$	0.1_{7}

- $\frac{1}{b} = 0.1_b$; $\frac{1}{b^2} = 0.01_b$
- $\frac{1}{b-1} = 0.\overline{1}_b$

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$base {\rightarrow}$	2	3	4	5	6	7
1/2	0.12	$0.\overline{1}_3$	0.24	$0.\overline{2}_{5}$	0.36	$0.\overline{3}_{7}$
1/3	$0.\overline{01}_2$	0.13	$0.\overline{1}_4$	$0.\overline{13}_{5}$	0.2_{6}	$0.\overline{2}_{7}$
1/4	0.01_2	$0.\overline{02}_3$	0.14	$0.\overline{1}_{5}$	0.13_{6}	$0.\overline{15}_{7}$
1/5	$0.\overline{0011}_{2}$	$0.\overline{0121}_{3}$	$0.\overline{03}_{4}$	0.15	$0.\overline{1}_{6}$	$0.\overline{1254}_{7}$
1/6	$0.0\overline{01}_{2}$	$0.0\overline{1}_{3}$	$0.0\overline{2}_{4}$	$0.\overline{04}_{5}$	0.16	$0.\overline{1}_{7}$
1/7	$0.\overline{001}_2$	$0.\overline{010212}_{3}$	$0.\overline{021}_{4}$	$0.\overline{032412}_{5}$	$0.\overline{05}_{6}$	0.17

$$\frac{1}{b} = 0.1_b$$
; $\frac{1}{b^2} = 0.01_b$

$$\frac{1}{b-1} = 0.\overline{1}_b$$

$$\frac{1}{b+1} = 0.\overline{0(b-1)}_b$$

$$(b+1)(b-1) = b^2 - 1$$

E3: Divisibility Tests

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E3

Contact Refs Let $d = d_m d_{m-1} \dots d_2 d_1 d_0$ be positive integer with base-10 digits d_i . Then d is divisible...

- by 2, 5, or 10 IFF d_0 is divisible by 2, 5, or 10 respectively; by 4 IFF the two-digit number d_1d_0 is divisible by 4; and by 8 IFF the three-digit number $d_2d_1d_0$ is divisible by 8.
- by 3 IFF $s = d_m + d_{m-1} + \cdots + d_1 + d_0$ is divisible by 3; and by 9 IFF s is divisible by 9.
- by 11 IFF $a = d_m d_{m-1} + \cdots \pm d_1 \mp d_0$ is divisible by 11.
- by 7 IFF the new integer $j = d_m d_{m-1} \dots d_2 d_1 + 5 d_0$ is divisible by 7. (Chika's Test)
- by 6 IFF it is divisible by both 2 and 3; and by 12 IFF it is divisible by both 3 and 4.

E3: Student Conjectures

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Conjecture

For any base b and any factor f that divides b, an integer d is divisibly by f if and only if the b^0 digit of d's base-b representation is divisible by f.

Conjecture

For any base b and any integer $d=d_md_{m-1}\dots d_2d_1d_0$ written in base-b notation, d is divisible by b-1 if and only if $s=d_m+d_{m-1}+\dots+d_2+d_1+d_0$ is divisible by b-1.

Conjecture

For any base b and any integer $d=d_md_{m-1}\dots d_2d_1d_0$ written in base-b notation, d is divisible by b+1 if and only if $a=d_m-d_{m-1}+\dots\pm d_2\mp d_1\pm d_0$ is divisible by b+1.



Why use this activity?

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Why

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- Conceptual learning (eg place value)
- Community, collaboration, inquiry, ...
- Depth and connectivity
- Decentering/perspective-taking



Thank You!

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Thank you!

Questions/Suggestions? Requests for links or resources?

bpkatzteach@gmail.com



References

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Thinking Q1 Q2

Extensions E1 E2 E3

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